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B. I. Rabinovich, G. G. Yefimenko and N. Ya. Dorozhkin

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INVESTIGATION OF THE DYNAMIC CHARACTERISTICS OF A SOLID BODY WITH A CYLINDRICAL CAVITY, PARTIALLY FILLED WITH A VISCOUS LIQUID

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ABSTRACT. The dynamic characteristics of a solid cylindrical body containing a cavity partially filled with a viscous liquid (water and glycerine mixture) are analyzed through boundary value problems. Equations are developed whose amplitude-frequency curves are shown to be very similar to experimental ones, so that they constitute a sufficiently accurate description of the motion of the model.

§1. Preliminary remarks and solution of boundary value problems. *Formulation of boundary value problems.* Let us use the equations of perturbed motion in one of the planes of symmetry of a solid body with a cavity partially filled with a viscous liquid, given in [5]: /11*

$$(m^\circ + m)\ddot{u} + \sum_{n=1}^{\infty} \lambda_n \ddot{s}_n = P;$$

$$(J^\circ + J)\ddot{\psi} + \sum_{n=1}^{\infty} (\lambda_{0n}\ddot{s}_n + \beta_{0n}\dot{s}_n) = M_G; \tag{1.1}$$

$$\mu_n(\ddot{s}_n + \beta_n\dot{s}_n + \omega_n^2 s_n) + \lambda_n\ddot{u} + \lambda_{0n}\ddot{\psi} + \beta_{0n}\dot{\psi} = 0 \qquad (n = 1, 2, \ldots),$$

where the point G is the metacenter of the body-liquid system.

All of the coefficients in (1.1) with the exception of β_n and β_{0_n}, are determined in the same manner as in the case of an ideal liquid [6]. The coefficients β_n and β_{0_n}, characterizing the effect of the viscosity of the liquid, are expressed by the functions Ψ, Ψ°, $\overset{\circ}{\phi}n$, $\overset{\circ}{\Omega}j$, Ωjn ($j = 1, 2, 3$; $n = 1, 2, \ldots$), which are the solutions of the boundary value problems formulated in [5]. The Reynolds number is determined by the formula $R = \omega r_0^2/j$.

These functions will be considered dimensionless relative to the radii of the cylinder $r_0(\phi_n, \phi_n^\circ, \Omega_{jn})$ and $r_0^2(\Psi, \Psi^\circ, \Omega_j^\circ)$.

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*Numbers in the margin indicate pagination of foreign text.

For ω we shall introduce the frequency which is characteristic for each of the boundary value problems and which can be identified with the n-th frequency of the vibrations of the liquid ω_n in the light of the concept in [5].

Let us move now to a cylindrical system of coordinates $oxr\theta$ with its origin in the center of the bottom of the cavity, where x and r are dimensionless coordinates relative to r_0, $0 \leqslant x \leqslant h/r_0 = \bar{h}$; $0 \leqslant r \leqslant 1$. We shall assume that $\phi_n = G_n(r,x) \sin\theta$, and introduce new notations and dimensionless unknown functions

$$\Psi^{\circ} = \varphi_0^{\circ};\quad \Psi = \varphi_0;\qquad \Omega_1^{\circ} = \Omega_{x0};\qquad \Omega_2^{\circ} = \Omega_{r0};\qquad \Omega_3^{\circ} = \Omega_{\theta 0};$$
$$\Omega_{1n} = \Omega_{xn};\qquad \Omega_{2n} = \Omega_{rn};\qquad \Omega_{3n} = \Omega_{\theta n}; \tag{1.2}$$

$$U_n(r,x);\qquad V_n(r,x);\qquad W_n(r,x);\qquad G_n^{\circ}(r,x);$$
$$\varphi_n^{\circ} = G_n^{\circ}(r,x)\sin\theta;\qquad \Omega_{rn} = -[U_n(r,x) + W_n(r,x)]\cos\theta; \tag{1.3}$$
$$\Omega_{\theta n} = [U_n(r,x) - W_n(r,x)]\sin\theta;\qquad \Omega_{xn} = -V_n(r,x)\cos\theta.$$

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The equations which define these functions are independent, and the boundary value problems in [5] become the following:

$$\Delta_1 G_n^{\circ} = 0;\qquad (\Delta_1 - i\mathrm{R})V_n = 0;$$
$$(\Delta_0 - i\mathrm{R})U_n = 0;\qquad (\Delta_2 - i\mathrm{R})W_n = 0. \tag{1.4}$$

$$\left.\begin{aligned}
&\frac{V_n}{r} - \frac{\partial U_n}{\partial x} + \frac{\partial W_n}{\partial x} + \frac{\partial G_n^{\circ}}{\partial r} = \begin{cases} -\left(\dfrac{\partial G_0}{\partial r} + x\right) & (n=0);\\[2mm] -\dfrac{\partial G_n}{\partial r} & (n=1,2,\ldots);\end{cases}\\
&\frac{\partial U_n}{\partial x} + \frac{\partial W_n}{\partial x} - \frac{\partial V_n}{\partial r} - \frac{G_n^{\circ}}{r}\begin{cases} \dfrac{G_0}{r} + x & (n=0);\\[2mm] \dfrac{G_n}{r} & (n=1,2,\ldots);\end{cases}\\
&\frac{\partial U_n}{\partial r} - \frac{\partial W_n}{\partial r} - \frac{2W_n}{r} + \frac{\partial G_n^{\circ}}{\partial x} = \begin{cases} -\dfrac{\partial G_0}{\partial x} + r & (n=0);\\[2mm] -\dfrac{\partial G_n}{\partial x} & (n=1,2,\ldots);\end{cases}
\end{aligned}\right| \tag{1.5}$$

$$\left.\begin{array}{l}\dfrac{\partial W_n}{\partial x}-\dfrac{\partial U_n}{\partial x}+\dfrac{V_n}{r}+\dfrac{\partial \overset{\circ}{G}_n}{\partial r}=0 \quad (n=0,1,2,\ldots);\\ \dfrac{\partial W_n}{\partial x}+\dfrac{\partial U_n}{\partial x}-\dfrac{\partial V_n}{\partial r}-\dfrac{\overset{\circ}{G}_n}{r}=0 \quad (n=0,1,2,\ldots);\\ \dfrac{\partial U_0}{\partial r}-\dfrac{\partial W_0}{\partial r}-\dfrac{2W_0}{r}+\dfrac{\partial \overset{\circ}{G}_0}{\partial x}=0;\end{array}\right| \text{на } L'. \tag{1.6}$$

Here

$$\Delta_m=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial x^2}-\frac{m^2}{r^2} \qquad (m=0,1,2);$$

L and L' are the lines of intersection of the wetted surface and the undisturbed free surface with the plane $\theta = \pi/2$.

In addition to conditions (1.5) and (1.6), the functions $\overset{\circ}{G}_n$, U_n, V_n and W_n ($n = 0, 1, 2, \ldots$) must satisfy the condition of regularity at $r = 0$.

Solution of boundary value problems for the functions U_n, V_n, W_n, $\overset{\circ}{G}_n$ *($n = 1, 2, \ldots$).* In the case under discussion, the function G_n has the form [4]

$$G_n(r, x) = \chi_n(x)\,\psi_n(r), \tag{1.7}$$

where

$$\chi_n(x)=\frac{\operatorname{ch}\xi_n x}{\xi_n \operatorname{sh}\xi_n h}; \qquad \psi_n(r)=\frac{J_1(\xi_n r)}{J_1(\xi_n)};$$

ξ_n — roots of the equation $J_1'(\xi) = 0$.

ξ_n are the roots of the equation $J_1'(\xi) = 0$.

We will seek the solution of the boundary value problem in the form /13

$$\overset{\circ}{G}_n=\sum_{j=1}^{2} G_n^{(j)}; \quad U_n=\sum_{j=1}^{2} U_n^{(j)}; \quad V_n=\sum_{j=1}^{2} V_n^{(j)}; \quad W_n=\sum_{j=1}^{2} W_n^{(j)}, \tag{1.8}$$

where the functions with the subscripts $j = 1$ and $j = 2$ exactly satisfy the boundary conditions at $r = 1$ and $x = 0$, respectively.

Selecting the appropriate partial solutions of equations (1.4) for functions with subscripts $j = 1$ and $j = 2$, we can express the desired solutions in the form of several series with indeterminate coefficients. The latter are determined with an accuracy up to the normalizing factor from the conditions of solvability of the systems of algebraic equations into which conditions (1.5) and (1.6) are transformed following the appropriate substitutions and expansions of the right-hand sides of the equations into Fourier series. As a result, we will have

$$U_n^{(1)} = \sum_{m=0}^{\infty} A_{nm} \frac{I_0(\alpha_n^m r)}{I_0(\alpha_n^m)} \cos \beta_m x; \qquad V_n^{(1)} = \sum_{m=1}^{\infty} B_{nm} \frac{I_1(\alpha_n^m r)}{I_1(\alpha_n^m)} \sin \beta_m x;$$

$$W_n^{(1)} = \sum_{m=0}^{\infty} C_{nm} \frac{I_2(\alpha_n^m r)}{I_2(\alpha_n^m)} \cos \beta_m x; \qquad G_n^{(1)} = \sum_{m=1}^{\infty} D_{nm} \frac{I_1(\beta_m r)}{I_1(\beta_m)} \sin \beta_m x; \tag{1.9}$$

$$U_n^{(2)} = A_n \frac{J_0(\xi_n r)}{J_0(\xi_n)} e^{-\varkappa_n x}; \qquad V_n^{(2)} = B_n \frac{J_1(\xi_n r)}{J_1(\xi_n)} e^{-\varkappa_n x};$$

$$W_n^{(2)} = C_n \frac{J_2(\xi_n r)}{J_2(\xi_n)} e^{-\varkappa_n x}; \qquad G_n^{(2)} = D_n \frac{J_1(\xi_n r)}{J_1(\xi_n)} \frac{\operatorname{sh} \xi_n (x - \bar{h})}{\operatorname{sh} \xi_n \bar{h}}.$$

Here

$$\alpha_n^m = \sqrt{\beta_m^2 + i\mathrm{R}_n}; \qquad \varkappa_n = \sqrt{\xi_n^2 + i\mathrm{R}_n};$$

$$\beta_m = \frac{\pi m}{\bar{h}}; \qquad \mathrm{R}_n = \frac{\omega_n r_0^2}{\nu}; \qquad \omega_n^2 = \frac{j\xi_n}{r_0} \operatorname{th} \xi_n \bar{h} \qquad (m = 0, 1, 2, \ldots).$$

The coefficients β_n, introduced in (1.1) depend only on D_{nm} and D_n, which are expressed by the formulas

$$D_{nm} = -\frac{1}{\Delta_{nm}} \left[\beta_m \left(\frac{\alpha_n^m I_0(\alpha_n^m)}{I_1(\alpha_n^m)} - 2 \right) \chi'_{nm} + \frac{\alpha_n^m I_1(\alpha_n^m)}{I_0(\alpha_n^m)} \chi_{nm} \right];$$

$$D_n = -\frac{1}{\varkappa_n \operatorname{ch} \xi_n \bar{h} - \xi_n \operatorname{sh} \xi_n \bar{h}};$$

$$\Delta_{nm} = -\frac{\beta_m \alpha_n^m I_0(\beta_m)}{I_1(\beta_m)} \left(\alpha_n^m - \frac{I_1(\alpha_n^m)}{I_0(\alpha_n^m)} \right) + \beta_m^2 \left(-2 + \frac{\alpha_n^m I_0(\alpha_n^m)}{I_1(\alpha_n^m)} \right) + (\alpha_n^m)^2$$

or, with large R_n, when $|\alpha_n^m| \approx |\varkappa_n| \sim \sqrt{\mathrm{R}_n}$

$$D_{nm} \approx \frac{I_1(\beta_m)(\chi_{nm} + \beta_m \chi'_{nm})}{\alpha_n^m \beta_m I'_1(\beta_m)}; \qquad D_n \approx -\frac{1}{\varkappa_n \operatorname{ch} \xi_n \bar{h}}, \tag{1.10}$$

where χ_{nm} and χ'_{nm} are the expansion coefficients of the functions χ_n and χ'_n into Fourier series according to the sines and cosines in the segment $0 \leqslant x \leqslant \bar{h}$; α_n^m and κ_n are the univalent roots of the corresponding functions, positive /14
in the first quadrant of the plane β_m^2, R_n or ξ_n^2, R_n.

Since $\varkappa_n \approx \alpha_n^m \approx \sqrt{\frac{R_n}{2}}(1+i)$ at large Reynolds numbers (for sufficiently small m), the solenoidal component of the velocity vector decreases exponentially from the bottom into the depths of the fluid, as well as with increasing distance from the walls in the radial direction. Solution (1.8), constituted from (1.9), satisfies (1.4) and at $\bar{h} \gg 1/\sqrt{R_n}$ it satisfies the boundary conditions (1.6) as well as (1.5), with the exception of a small area (on the order of $1/\sqrt{R_n}$) of the corner point of line L.

Solution of boundary value problems for functions U_0, V_0, W_0 and G_0°. We shall seek the solution of the boundary value problem in the form of a sum of the three partial solutions

$$G_0^\circ = \sum_{j=1}^{3} G_0^{(j)}; \quad U_0 = \sum_{j=1}^{3} U_0^{(j)}; \qquad V_0 = \sum_{j=1}^{3} V^{(j)}; \quad W_0 = \sum_{j=1}^{3} W_0^{(j)}, \tag{1.11}$$

where the functions with the subscripts j = 1 and j = 2 satisfy precisely the boundary conditions for r = 1 and x = 0, respectively.

We will select the functions with subscript j = 3 so that all three solutions will satisfy exactly conditions (1.6) in the sum. For a cavity in the form of an upright circular cylinder, we will have [4]

$$G_0 = -xr + 2\sum_{n=1}^{\infty} \frac{K_n(x)\psi_n(r)}{\xi_n^2 - 1}. \tag{1.12}$$

Here

$$K_n(x) = \frac{\operatorname{ch} \xi_n x - 2\operatorname{ch} \xi_n (x - \bar{h})}{\xi_n \operatorname{sh} \xi_n \bar{h}}; \qquad \psi_n = \frac{J_1(\xi_n r)}{J_1(\xi_n)}.$$

Employing the functions (1.12) with the aid of the method described, we obtain the following solutions for $U_0^{(j)}$, $V_0^{(j)}$, $W_0^{(j)}$, $G_0^{(j)}$ $(j = 1, 2, 3)$:

$$U_0^{(1)} = \sum_{m=0}^{\infty} A_{0m} \frac{I_0(\alpha_0^m r)}{I_0(\alpha_0^m)} \cos \beta_m x; \quad V_0^{(1)} = \sum_{m=1}^{\infty} B_{0m} \frac{I_1(\alpha_0^m r)}{I_1(\alpha_0^m)} \sin \beta_m x;$$

$$W_0^{(1)} = \sum_{m=0}^{\infty} C_{0m} \frac{I_2(\alpha_0^m r)}{I_2(\alpha_0^m)} \cos \beta_m x; \quad G_0^{(1)} = \sum_{m=1}^{\infty} D_{0m} \frac{I_1(\beta_m r)}{I_1(\beta_m)} \sin \beta_m x;$$

$$U_0^{(2)} = \sum_{n=1}^{\infty} A'_n \frac{J_0(\xi_n r)}{J_0(\xi_n)} e^{-\varkappa_n x}; \quad V_0^{(2)} = \sum_{n=1}^{\infty} B'_n \frac{J_1(\xi_n r)}{J_1(\xi_n)} e^{-\varkappa_n x}; \tag{1.13}$$

$$W_0^{(2)} = \sum_{n=1}^{\infty} C'_n \frac{J_2(\xi_n r)}{J_2(\xi_n)} e^{-\varkappa_n x}; \quad G_0^{(2)} = \sum_{n=1}^{\infty} D'_n \frac{J_1(\xi_n r)}{J_1(\xi_n)} \frac{\operatorname{sh} \xi_n (x - \bar{h})}{\operatorname{sh} \xi_n \bar{h}};$$

$$U_0^{(3)} = \sum_{n=1}^{\infty} \tilde{A}_n \frac{J_0(\xi_n r)}{J_0(\xi_n)} e^{-\varkappa_n(\bar{h}-x)}; \quad V_0^{(3)} = \sum_{n=1}^{\infty} \tilde{B}_n \frac{J_1(\xi_n r)}{J_1(\xi_n)} e^{-\varkappa_n(\bar{h}-x)};$$

$$W_0^{(3)} = \sum_{n=1}^{\infty} \tilde{C}_n \frac{J_2(\xi_n r)}{J_2(\xi_n)} e^{-\varkappa_n(\bar{h}-x)}; \quad G_0^{(3)} = \sum_{n=1}^{\infty} \tilde{D}_n \frac{J_1(\xi_n r)}{J_1(\xi_n)} \frac{\operatorname{sh} \xi_n x}{\operatorname{sh} \xi_n \bar{h}},$$ /15

where the coefficients D_{0m}, D'_n are expressed by the formulas

$$D_{00} = 0; \quad D_{0m} \approx \frac{2 I_1(\beta_m)}{\alpha_0^m \beta_m I'_1(\beta_m)} \sum_{n=1}^{\infty} \frac{\beta_m K'_{nm} + K_{nm}}{\xi_n^2 - 1};$$

$$D'_n = \frac{2}{\xi_n^2 - 1} \frac{1 - 2 \operatorname{ch} \xi_n \bar{h}}{\xi_n \operatorname{sh} \xi_n \bar{h} - \varkappa_n \operatorname{ch} \xi_n \bar{h}};$$

Here K_{nm} and K'_{nm} are the expansion coefficients of the functions $K_n(x)$ and $K'_n(x)$ into Fourier series according to the sines and cosines; they have the form

$$K_{nm} = -\frac{\beta_m}{\xi_n^2} K'_{nm}; \quad K'_{nm} = \frac{2\xi_n \{[(-1)^m + 2] \operatorname{ch} \xi_n \bar{h} - [2(-1)^m + 1]\}}{\bar{h} \operatorname{sh} \xi_n \bar{h} (\xi_n^2 + \beta_m^2)}.$$

For the coefficient $\tilde{D}_n$, after transformations and the application of summation formulas [3], we obtain the following expression, accurate with a precision up to terms on the order of $1/\sqrt{R_n}$,

$$\widetilde{D}_n = -\frac{2\,\mathrm{sh}\,\xi_n\bar{h}}{\xi_n(\xi_n^2-1)\varkappa_n\,\mathrm{ch}\,\xi_n\bar{h}}\left\{\xi_n^2+1+\frac{\xi_n(2\,\mathrm{ch}\,\xi_n\bar{h}-1)}{\mathrm{sh}\,\xi_n\bar{h}\,\mathrm{ch}\,\xi_n\bar{h}}-\frac{2\xi_n\bar{h}}{\mathrm{sh}\,\xi_n\bar{h}}+\right.$$

$$\left.+2\xi_n^2\sum_{\substack{p=1\\p\neq n}}^{\infty}\left[\frac{1}{\xi_p^2-\xi_n^2}+\frac{\xi_n}{\xi_p}\,\frac{1-2\,\mathrm{ch}\,\xi_p\bar{h}+2\,\mathrm{ch}\,\xi_n\bar{h}-\mathrm{ch}\,\xi_p\bar{h}\,\mathrm{ch}\,\xi_n\bar{h}}{(\xi_p^2-\xi_n^2)\,\mathrm{sh}\,\xi_p\bar{h}\,\mathrm{sh}\,\xi_n\bar{h}}\right]\right\}; \tag{1.14}$$

§2. Determination of the parameters δ_n and δ_{0n}. Let us introduce into the discussion the logarithmic decrement of the vibrations of the fluid δ_n and the parameter δ_{0n}, related to the coefficients β_n and β_{0n} by the relationships

$$\delta_n=\frac{\pi\beta_n}{\omega_n};\qquad \delta_{0n}=\frac{\pi\beta_{0n}}{\mu r_0\omega_n}. \tag{2.1}$$

Here

$$\mu_n=\frac{jN_n^2}{r_0\omega_n^2}=\frac{\pi(\xi_n^2-1)}{2\xi_n^3\,\mathrm{th}\,\xi_n\bar{h}};$$

$$\omega_n^2=\frac{j\xi_n}{r_0}\,\mathrm{th}\,\xi_n\bar{h};\qquad N_n^2=\int_\Sigma\psi_n^2ds.$$

To determine δ_n and δ_{0n}, let us use the formulas in [5], limiting ourselves to the terms on the order of $1/\sqrt{R_n}$ and considering dimensionless the coordinates, the scalar and vector potentials.

The corresponding expressions in a cylindrical system of coordinates can be written, with consideration of (1.3), in the form /16

$$\delta_n=\frac{\pi}{N_n^2}\,\mathrm{Im}\left[\int_\Sigma\left(\frac{\partial\mathring{G}_n}{\partial x}+\frac{\partial U_n}{\partial r}-\frac{\partial W_n}{\partial r}-\frac{2W_n}{r}\right)\psi_n\sin\theta ds-\right.$$

$$\left.-\frac{r_0\omega_n^2}{j}\int_\Sigma\mathring{G}_n\psi_n\sin\,\theta ds\right]; \tag{2.2}$$

$$\delta_{0n}=-\frac{\omega_n^2\pi r_0}{jN_n^2}\,\mathrm{Im}\int_\Sigma\mathring{G}_0\psi_n\sin\theta ds.$$

Substituting in (2.2) the corresponding expressions from (1.9), (1.13) as well as (1.10) and (1.14), after several transformations are summed by the subscript m [3], with an accuracy up to terms on the order of $1/\sqrt{R_n}$, we will have

$$\delta_n = \frac{\pi}{\sqrt{2R_n}}\left(\frac{\xi_n^2+1}{\xi_n^2-1} + \frac{2\xi_n}{\operatorname{sh} 2\xi_n\bar{h}}\right); \tag{2.3}$$

$$\delta_{0n} = -\frac{2\pi \operatorname{th}^2 \xi_n\bar{h}}{\sqrt{2R_n}(\xi_n^2-1)}\left[\xi_n^2 + 1 + \frac{2(1-\bar{h})\xi_n}{\operatorname{sh}\xi_n\bar{h}} - \frac{\xi_n}{\operatorname{sh}\xi_n\bar{h}\operatorname{ch}\xi_n\bar{h}} + \right.$$

$$\left. + 2\xi_n^2 {\sum_{p=1}^{\infty}}' \frac{1}{\xi_p^2-\xi_n^2}\left(1 + \frac{\xi_n}{\xi_p}\,\frac{1+2\operatorname{ch}\xi_n\bar{h} - 2\operatorname{ch}\xi_p\bar{h} - \operatorname{ch}\xi_n\bar{h}\operatorname{ch}\xi_p\bar{h}}{\operatorname{sh}\xi_n\bar{h}\operatorname{sh}\xi_p\bar{h}}\right)\right]. \tag{2.4}$$

In the boundary cases, formulas (2.3) and (2.4) change to the following: for a deep fluid ($\bar{h} \gg 1$)

$$\delta_n = \frac{\pi}{\sqrt{2R_n}}\,\frac{\xi_n^2+1}{\xi_n^2-1}; \tag{2.5}$$

$$\delta_{0n} = -\frac{2\pi}{\sqrt{2R_n}}\,\frac{\xi_n^2}{\xi_n^2-1}\left[1 + 2\sum_{p=1}^{\infty}\frac{1}{\xi_p(\xi_p+\xi_n)}\right]; \tag{2.6}$$

for a shallow fluid ($\bar{h} \ll 1$)

$$\delta_n = \frac{\pi}{\sqrt{2R_n\bar{h}}} \quad (2.7); \qquad \delta_{0n} = -\frac{2\pi}{\sqrt{2R_n}}\,\frac{\xi_n^2\bar{h}}{\xi_n^2-1}. \tag{2.8}$$

Figure 1 shows the coefficients $\bar{\delta}_n = \delta_n\sqrt{R_1}$ and $\bar{\delta}_{0n} = \delta_{0n}\sqrt{R_1}$ as functions of the level of fullness $\bar{h}$, calculated with the aid of (2.3) and (2.4) for the first four forms of vibration ($n = 1, 2, 3, 4$).

At $\bar{h} \to 0$, formulas (2.3) and (2.7) lose the physical meaning that is related to the proximity of the statement of the problem and the method of solution.

§3. Analysis of the numerical results and comparison with experiment. We will limit ourselves to a consideration of the case of a deep fluid ($\bar{h} > 1$); equations (1.1) will deal only with the first note of the vibration ($n = 1$). /17

Dropping the subscript 1 in the generalized coordinates, we can write (1.1) thus:

$$
\begin{aligned}
&(m^\circ + m)\ddot{u} + \lambda_1 \ddot{s} = P;\\
&(J^\circ + J)\ddot{\psi} + \lambda_{01}\ddot{s} + \beta_{01}\dot{s} = M_G;\\
&\mu_1(\ddot{s} + \beta_1\dot{s} + \omega_1^2 s) + \lambda_1\ddot{u} + \lambda_{01}\ddot{\psi} + \beta_{01}\dot{\psi} = 0.
\end{aligned}
\qquad (3.1)
$$

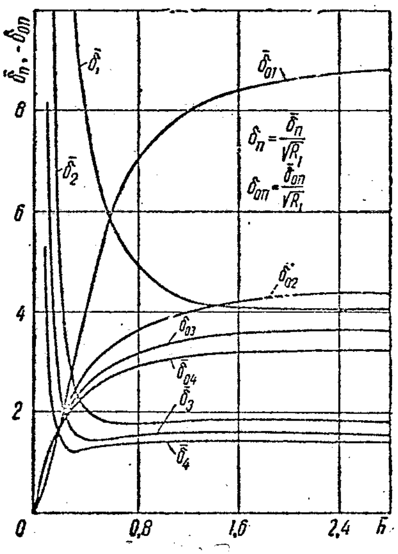

Figure 1

To estimate the effect of the additional dissipative terms on the dynamic characteristics of the system, we performed calculations of the parameters δ_1 and δ_{01} using (2.5) and (2.6). These same parameters were estimated independently on the basis of a comparison of the theoretical and experimental frequency characteristics of the "body-fluid" system with translational and rotational displacements.

In carrying out the experiments, we used as a model a cylindrical cavity with a diameter of 350 mm, filled with a mixture of water and glycerine to the level h = 295 mm, with an axis of suspension passing through the metacenter of the "body-fluid" system. Calculations were performed with consideration of the concrete parameters of the cavity and the suspension elements.

The independent translational and angular displacements of the model are described by the following systems of equations:

$$
\begin{aligned}
&\ddot{u} + \beta_u\dot{u} + \omega_u^2 u + \ddot{\zeta} = P; \quad \ddot{\zeta} + \beta_\zeta\dot{\zeta} + \omega_\zeta^2\zeta + a_{\zeta u}\ddot{u} = 0;\\
&\ddot{\psi} + \beta_\psi\dot{\psi} + \omega_\psi^2\psi + a_{\psi\zeta}\ddot{\zeta} + a'_{\psi\zeta}\dot{\zeta} = M;\\
&\ddot{\zeta} + \beta_\zeta\dot{\zeta} + \omega_\zeta^2\zeta + a_{\zeta\psi}\ddot{\psi} + a'_{\zeta\varphi}\dot{\psi} = 0.
\end{aligned}
\qquad (3.2)
$$

Here

$$\zeta = \frac{\lambda_1}{m^\circ + m} s; \qquad \beta_\zeta = \frac{\delta_1 \omega_\xi}{\pi}; \quad a_{\zeta u} = \frac{\lambda_1^2}{\mu_1 (m^\circ + m)};$$

$$a_{\psi\zeta} = \frac{\lambda_{01}}{\lambda_1} \frac{m^\circ + m}{J^\circ + J}; \qquad a_{\zeta\psi} = \frac{\lambda_{01}\lambda_1}{\mu_1 (m^\circ + m)}; \tag{3.3}$$

$$a'_{\psi\zeta} = \frac{\beta_{01}}{\lambda_1} \frac{m^\circ + m}{J^\circ + J}; \qquad a'_{\zeta\psi} = \frac{\beta_{01}\lambda_1}{\mu_1 (m^\circ + m)},$$

For symmetry, the designations are changed somewhat in comparison with (3.1).

In [1] there is an empirical formula for calculating the logarithmic decrement of the vibrations of the fluid at n = 1. In the case where $\bar{h} > 1$, this formula has the form /18

$$\delta_1 = \frac{1{,}84\pi}{\sqrt{R_1}} \tag{3.4}$$

With an accuracy up to a constant factor $\sqrt{2}$ it agrees with (2.5) (at n = 1) and with the corresponding formula given in [7].

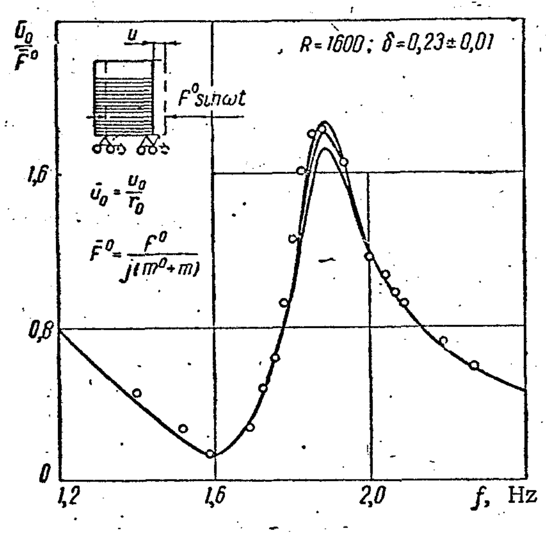

Figure 2

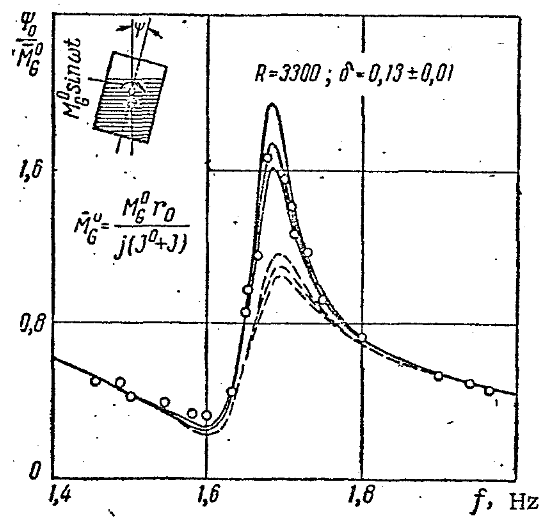

Figure 3

The experiment showed that for numbers $R_1 < 10^4$ the relationship (3.4) gives reduced values of δ_1 and a correction must be introduced in it.

In calculating the amplitude-frequency characteristic curves of the systems (3.2), the coefficients β_u, β_ψ, ω_u^2, ω_ψ^2 and J° were determined experimentally, and logarithmic decrement δ_1 by formula (3.4) with the suitable correction.

The frequency of the fluid ω_s and the coefficients λ_1, λ_{01}, μ_1 and J, included in expressions (3.3) were calculated according to the formulas for an ideal fluid [4] and checked by means of individual experiments according to the method described in [2].

Thus, of all the coefficients in (3.2), it was necessary to verify in the final stage of the experiments only the value β_{01}, calculated with the aid of (2.1) and (2.6), which was done on the basis of a comparison of the theoretical and experimental amplitude-frequency characteristic curves in the vicinity of the natural frequency of the system, where the effect of the coefficient β_{01} is maximal.

Figure 2 and Figure 3 show the theoretical (solid curve) and experimental (circles) amplitude-frequency curves of the translational and angular displacements of the model. The theoretical curves were calculated for several values of the decrement δ_1, in order to allow for possible errors in determining the viscosity of the fluid. It is apparent from the figures that the theoretical amplitude curves are very close to the experimental ones. This allows us to conclude that the systems of equations (3.2) are a sufficiently accurate description of the motion of the model, and (2.6) gives a result which is close to the actual one, at least in the range of R_1 numbers under consideration. For /19
an illustration of the effect of the coefficient β_{01} on the dynamic curve of the system in Figure 3 we have used a dashed line to represent the amplitude characteristic curve calculated on the assumption that $\beta_{01} = 0$.

We can show that the value of the correction to the dynamic coefficient of amplification of the model at the natural frequency of the model, introduced by additional terms in (3.2), dependent on β_{01}, is determined by the value of the coefficient

$$\Delta_1 = \frac{2r_0\delta_{01}\lambda_{01}}{\delta_1(J^\circ + J)}, \qquad (3.5)$$

which is comparable with unity.

According to (2.4), (2.6) and (2.8), the parameter δ_{01} has the property of fixed sign. Therefore the sign of coefficient Δ_1 can be either positive or negative depending on the sign of λ_{01}. In accordance with this, the correction of the dynamic coefficient of amplification of the open system will be positive or negative, and the solid curve in Figure 3 will run above or below the dashed one. Thus, the disregard of the terms dependent on parameter β_{01} in the equations of excited motion (3.1) in certain cases may not lead to a "margin" of stability.

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